

Did planet formation begin inside persistent gaseous vortices?

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Abstract. We explore here the idea, reminiscent in some respect of Von Weizsäcker's (1944) and Alfvén's (1976) outmoded cosmogonies, that long-lived vortices in a turbulent protoplanetary nebula can capture large amount of solid particles and initiate the formation of planets. Some puzzling features of the solar system appear as natural consequences of our simple model:

- The captured mass presents a maximum near Jupiter's orbit.

- Outside this optimal orbit, the collected material, mainly composed of low density particles, sinks deeply into the vortices and rapidly collapses into massive bodies at the origin of the solid core of the giant planets.

- Inside this orbit, by contrast, the high density particles are preferentially selected by the vortices and assembled by local gravitational instabilities into planetesimals, massive enough to be released by the vortices and to grow later, in successive collisions, to form the terrestrial planets.

Key words: Planet formation - solar system - vortices - accretion disks

Planets are thought to be formed from the dust grains embedded in a gaseous disk, probably like observed around most young low-mass stars: the Protoplanetary Nebula. It is likely that this by-product of the sun formation was also, for a while, a turbulent accretion-disk. During this stage the star completes its accretion and the disk spreads outward with the angular momentum (justifying the repartition of mass and momentum between sun and planets). In the meantime the dust grains are submitted to a turbulent diffusion (due to the gas motions) which speeds up their growth and enables to explain the chemical composition of some meteorites (Morfill, 1983). Due to collisional fragmentation, this turbulent coagulation stalls for centimeter-sized particles in a highly turbulent nebula (Weidenschilling, 1984) and for meter-sized particles in a weakly turbulent nebula (Weidenschilling and Cuzzi, 1993), while gravitational binding becomes effective only in the kilometer range. So, it is commonly thought that the solid material decouples from the gas only after some turbulence decayed, in a two stage process:

- (i) settling of the dust grains toward the mid-plane of the gaseous disk;

- (ii) gravitational collapse of the resulting layer of sediment (when dense enough) into numerous kilometer-sized bodies, the so called "planetesimals".

Then, as suggested by the cratering of the present planets, gravitationally bounded bodies grow by the accumulation of planetesimals in successive collisions; this stage of the planet growth is, indeed, reproduced by a number of dynamical models (Safronov 1969; Barge and Pellat 1991, 1993).

However the above scenario faces two major difficulties.

- (1) The solid cores of the giant planets must be formed in less than some 10^6 years, in order for the gas to be captured before being swept away (Safronov 1969; Strom *et al.* 1993) during the sun's T-Tauri phase; with a reasonable density of solid material, this is difficult to achieve by planetesimal accumulation (Safronov 1969; Wetherill 1988), especially for the outermost planets.

- (2) The formation of the planetesimals themselves is not clearly understood. Indeed the gas, which is supported by a radial pressure gradient, rotates at slightly less than the local Keplerian speed. The resulting velocity difference ΔV between the sediment layer and the overlying gas induces shear turbulence that prevents the layer from settling to the density required for gravitational instability (Weidenschilling and Cuzzi 1993; Cuzzi *et al.* 1993).

Both difficulties are solved by the present model, in which the particles, once settled in the nebula mid-plane, are captured and concentrated into long-lived vortices.

Such vortices may be maintained by specific instability mechanism (Dubrulle 1993), but more generally emerge from random turbulence in rotating shear flows. While three-dimensional eddies are quickly damped by energy cascade toward small scales, two-dimensional turbulence persists without energy dissipation, forming instead larger and larger vortices until a steady solitary vortex is formed. Striking examples are the persistent atmospheric vortices in the giant planets (Ingersoll 1990), like Jupiter's Great Red Spot. This phenomenon can be reproduced in laboratory experiments (Antipov *et al.* 1986; Sommeria *et al.* 1988; Nezlin and Snezhkin 1993), and explained in terms of statistical mechanics of two-dimensional turbulence (Sommeria *et al.* 1991; Miller *et al.* 1992; Michel and Roberts 1994). Observations of accretion-disks around black-

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holes (Abramowicz *et al.* 1992) or T-tauri stars could also indicate the presence of such organized vortices.

In this letter, as we focus on particle trajectories, fluid dynamics will be discussed at an heuristic level, sufficient to justify a simple vortex model. Particle motions are referred to a cartesian frame, in which x and y stand for the azimuthal position and the radial displacement, respectively, rotating around the sun at the Keplerian angular velocity $\Omega = \Omega_0 r^{-3/2}$ (Ω_0 is the Earth's velocity for $r = 1$, in astronomical unit, AU). The y axis is directed outward, and the x axis along the orbital motion, which has then a clockwise (negative) rotation.

Under the standard assumption of hydrostatic balance in the thickness H of the nebula ($H \simeq C_S/\Omega$, where C_S is the sound speed), the dynamical problem is two-dimensional. Further, neglecting pressure forces, the simplest flow is a set of circular orbits with Keplerian azimuthal velocity ($V_x = -3\Omega y/2$ and $V_y = 0$). The small vortices, possibly rising in this flow with scale $R < H$ and typical vorticity Ω , have a velocity of the order of ΩR which is less than the sound speed; so, they can be considered as incompressible. Then, vortices spinning like the shear flow are robust and merge one another. (while those with opposite sign are laminated by the shear) (Marcus 1990; Dowling and Ingersoll 1989). The process of vortex growth ends up when the Mach number reaches unity (i.e) $R \simeq H$, beyond which energy losses by sound waves become prohibitive. Finally, the vortex structure should evolve as to minimize the pressure effects responsible for these losses, the streamlines fitting at best with the free-particle trajectories.

An obvious solution is a set of Keplerian ellipses with the same semi-major axis but different eccentricities, corresponding in our rotating frame, to concentric epicycles ($V_x = -2\Omega y$, $V_y = \Omega x/2$); this is a steady solution of the fluid equations with uniform pressure. A correspondence between epicyclic motion and vortex flow was used first by Von Weizsäcker (1944); it appears also in the dynamics of non-axisymmetric planetary rings where fluid streamlines can coincide with particle trajectories and describe the ring shapes (Borderies *et al.* 1982). We assume a simple matching of this "epicyclic flow" with the azimuthal Keplerian flow at large distances (see Fig.1):

$$\begin{cases} V_x = -\frac{3}{2}\Omega y - \frac{1}{2}\Omega x e^{-\frac{x^2+y^2}{2R^2}} \\ V_y = \frac{1}{2}\Omega x e^{-\frac{x^2+y^2}{2R^2}} \end{cases}.$$

The characteristic size R (or "radius") of this vortex is limited to the thickness H of the nebula as discussed above. Its decay time under persistent three-dimensional turbulence can be estimated, using the classical turbulent viscosity for accretion-disks $\nu_t = \alpha C_S^2/\Omega$ with $\alpha = 10^{-3}$ (following current nebula models $10^{-4} < \alpha < 10^{-2}$). The corresponding energy decay time τ_D is then estimated by dividing the rate of viscous energy dissipation by the vortex kinetic energy. This yields $\Omega\tau_D = 6.5/\alpha$, so that τ_D is about 500 rotation periods $2\pi/\Omega$. For the Great Red Spot of Jupiter or Dark Oval of Neptune, the ratio between the estimated friction time (~ 10 years) and rotation time (a few days) is similar. We then similarly expect that, at a given distance from the sun, successive mergings have time to produce a unique vortex or at most very few of them. By contrast, vortices with sufficient radial separation (a few times their size $R \sim H$) cannot merge, so that we finally expect a set of independent isolated vortices with radial interval scaling like the nebula thickness. Since H scales with a power law of the distance to the sun ($r^{5/4}$ in the

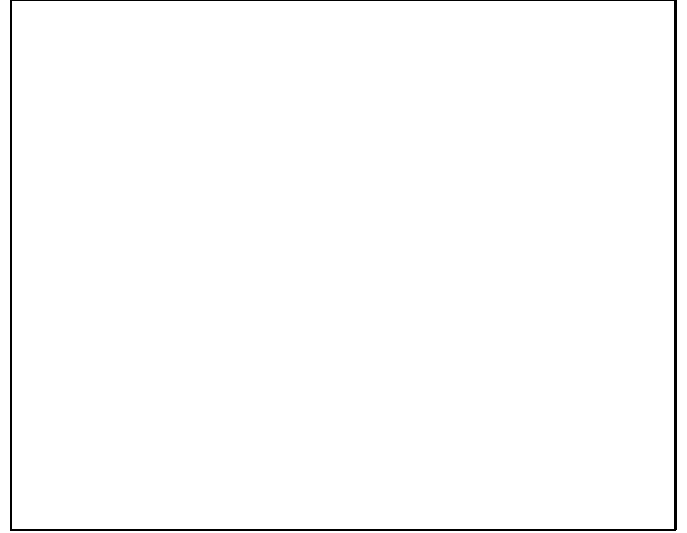


Fig. 1. Trajectories of the particles captured in a gaseous vortex, sketched by the separatrix (dashed line) between open and closed streamlines. The particles penetrate into the vortex and spiral inward toward its center; they tend to reach purely epicyclic motion with a transient behaviour strongly dependent on the friction parameter: light particles ($\tau_S = 0.05$ in case (a)) remain near the edge of the vortex, whereas heavy ones ($\tau_S = 3$ in case (b)) first sink deeply into the inner regions. It must be noted that, for clarity of the figure, the ordinates have been expanded by a factor of two.

standard model considered below), this is consistent with an approximate geometric progression of the planetary positions.

The particles embedded in the gas of the nebula are submitted to a friction drag whose expression depends on the mean-free-path of the gas molecules relative to the particle size. For decimetric particles (and beyond 2AU from the sun), mean-free-path exceeds size and the drag reaches the Epstein regime. The motion equations of the particles, submitted to the sun attraction, Coriolis force and friction drag, then read:

$$\begin{cases} \frac{dv_x}{dt} = -2\Omega v_y - \frac{1}{t_S}(v_x - V_x) \\ \frac{dv_y}{dt} = 3\Omega^2 y + 2\Omega v_x - \frac{1}{t_S}(v_y - V_y) \end{cases},$$

where $t_S = \rho_d s / (\rho_{gas} C_S)$ is the stopping-time for a spherical particle with radius s and density ρ_d in a gas with density ρ_{gas} . The dynamical evolution depends on the single non-dimensional friction parameter $\tau_S = \Omega t_S$, that is on the particle mass/area ratio: (i) the lightest particles ($\tau_S \ll 1$) come at rest rapidly with the gas and travel with the local flow; (ii) the heaviest particles ($\tau_S \gg 1$) cross the vortex with a keplerian motion nearly unaffected by the friction drag.

In the intermediate range of τ_S , a numerical integration of the equations shows that a particle can be captured by the vortex if its impact parameter (initial distance to the x axis) is sufficiently small (Fig.1); otherwise it is dragged by the flow. The corresponding critical impact parameter η_c can be fitted by the function (see Fig.2)

$$f(\tau_S) = \frac{\eta_c}{R} = \frac{A \tau_S^{1/2}}{\tau_S^{3/2} + B},$$

where $A \simeq 2.4$ and $B \simeq 2.2$. This function reaches a maximum when $\tau_S \simeq 1$, and reduces to the power laws $\tau_S^{1/2}$ and τ_S^{-1} ,

in the limits of the light and heavy particles, respectively. As the approach velocity ($3\eta\Omega/2$) only depends on the impact parameter, the mass capture rate is straightforwardly:

$$\frac{dM_{capt}}{dt} = \frac{3}{2}\sigma R^2\Omega f^2(\tau_S)$$

where σ is the mean surface density of nebular solid material.



Fig. 2. Non dimensional capture cross-section of the vortex as a function of the friction parameter ($\tau_S = \Omega t_S$). Filled squares represent the values obtained by successive numerical integrations. The dashed line is the function $f(\tau_S) = A\tau_S^{1/2}/(\tau_S^{3/2} + B)$ fitting at best the dependance $\eta_c(\tau_S)/R$. The curve distinguishes between two different types of trajectories: below it, they wind up round the vortex center and correspond to a capture; whereas above it, they leave the vortex zone and correspond to simple crossing.

We now evaluate how the capture rate depends on the distance from the sun, choosing a standard model of nebula (Cuzzi *et al.* 1993) in which the surface densities (both for gas and for particles) and the temperature are the decreasing power-laws $r^{-3/2}$ and $r^{-1/2}$, respectively; at 1 AU the densities are set to 1700gcm^{-2} for the gas and to 20gcm^{-2} for the particles, whereas the temperature is assumed to be 280 K. Consequently, the thickness of the nebula H ($\simeq R$), approximately 0.04 AU near Earth's orbit, increases as $r^{5/4}$.

Further, as to get the essence of our capture mechanism, it is sufficient to assume that all the particles have the same density $\rho_d = 2\text{gcm}^{-3}$ (the density of a composite rock-ice material) and the same size $s_* = 40\text{cm}$ (a typical prediction in a gravitationally stable layer of sediment (Weidenschilling and Cuzzi 1993), inside which fragmentation is less effective than in a fully turbulent accretion-disk). The friction parameter, which writes $\tau_S = 2\rho_d s/\sigma_{gas}$, increases as $r^{3/2}$. As a result the capture rate, proportional to $f^2(\tau_S)$, is optimum at the distance r_* from the sun for which the function $r f^2$ is maximum. With our numerical values this optimum is reached when $r_* \simeq 7.5\text{ AU}$, that is in between the present Jupiter's and Saturn's orbits, explaining the predominance of these two planets. The mass collected (at constant rate) reaches typical planetary values after a time Δt corresponding to 500 revolutions of the vortex (see table I). The decrease of the predicted masses at

Table 1. Amount of captured mass

$r\text{ (A.U.)}$	$\Delta t\text{ (yrs)}$	τ_S	$M_{capt}(M_\oplus)$	$M_{core}(M_\oplus)^a$
1	$5.00\ 10^2$	0.09	0.6	—
2	$1.41\ 10^3$	0.27	3.2	—
5	$6.00\ 10^3$	1.05	16.0	15 – 30
10	$1.45\ 10^4$	2.97	18.0	16 – 23
20	$4.20\ 10^4$	8.42	7.8	11 – 13
30	$8.25\ 10^4$	15.46	3.8	14 – 16

Note: (a) Classical estimation of the amount of high-Z material contained in the giant planets (Pollack 1985); notice that these values are still under debate and, as recently proposed (Guillot *et al.* 1994), could be significantly smaller.

large distance seems a bit too strong, when compared to the estimated masses (Pollack 1985) of heavy elements contained in the giant planets; in fact, it would be reduced by accounting for a dispersion in particle size and density. On the other hand, comparison with the masses of the terrestrial planets has been discarded as requiring the further modelling of collisional accumulation.

The above calculations implicitly assume that the particles are continuously renewed near the vortex orbit. This occurs due to the inward drift under the systematic drag associated with the velocity difference ΔV between gas and particles. This drift, indeed, which reaches its optimum value ΔV for $\tau_S \sim 1$ (that is near Jupiter's orbit), results in a mass flux exceeding easily the capture rate (We have also introduced this drift in the expression for the gas velocity V_x and check that it has no influence on the capture cross section).

This two dimensional capture mechanism adds to the vertical settling toward the nebula midplane, which is known to form a particle sublayer whose typical thickness is $H_p \simeq 10^{-3}H$ (Cuzzi *et al.* 1993). It results, inside the vortices, in an increasing surface density σ_{vort} and in a stronger volume density σ_{vort}/H_p which reaches much more easily the Roche threshold for gravitational instability. In terms of the particle velocity dispersions $C_p \sim \Omega H_p$ the criterion for instability to occur reads:

$$C_p \leq \frac{\pi G \sigma_{vort}}{\Omega},$$

where G is the gravitational constant. In the absence of any surface density enhancement ($\sigma_{vort} = \sigma$), this velocity threshold is very low, 20cms^{-1} in our nebula model, and is easily exceeded by any residual turbulence. Indeed, according to the classical model of turbulent accretion-disk, the velocity dispersion $C_p = \alpha C_S$ (with $\alpha = 10^{-3}$) is of the order of 2ms^{-1} at 1 AU. Even with an initially laminar nebula, a minimal velocity dispersion (Weidenschilling and Cuzzi 1993; Cuzzi *et al.* 1993) $C_p = 2\Delta V/Re^*$ (where $Re^* \sim 100$) would be generated by the turbulent shear between the particle sublayer (considered as heavy fluid) and the overlying gas. This velocity difference ΔV between gas and particles is approximately 60ms^{-1} at any distance from the sun and the resulting velocity dispersion (several ms^{-1}) is sufficient to inhibit gravitational instability.

By contrast, inside the vortices, the surface density is increased by several order of magnitude in some ten rotation

periods, so that gravitational instabilities become much easier and rapidly gather the material into planetesimals.

The fate of these planetesimals (nearly insensitive to the gas friction) will, in fact, strongly depends on the friction parameter τ_S of the particles they formed from, that is on the distance from the sun. Indeed preliminary computations indicate that:

(1) Inside Jupiter's orbit ($\tau < 1$), particle concentration occurs in an annular region at the vortex periphery, as clearly seen from the dotted trajectory of Fig.1; the resulting planetesimals have wide epicyclic oscillations and are quickly released from the vortex. Afterward the growth of the terrestrial planets would proceed following the "standard" collisional history (Safronov 1969; Barge and Pellat 1991, 1993).

(2) Outside Jupiter's orbit ($\tau > 1$), the trapped particles deeply sink into the central vortex region (solid trajectory of Fig.1) and reach an epicyclic, nearly ballistic motion; once formed, planetesimals remain along similar slowly-evolving orbits, until they collapse into a single body, massive enough to form a Giant Planet after the capture of the surrounding gas (Safronov 1969; Pollack 1985).

The direction of planetary rotation depends on the subsequent phases of dust contraction and gas accretion, and the result is far from obvious (Dones and Tremaine, 1993; Coradini et al. 1989). However, it is straightforward to show that the angular momentum of a swarm of particles contracting under self-gravity and inelastic collisions, when referred to its center of mass, is conserved, in the standard inertial frame of reference; then, simple calculations indicate that this angular momentum is prograde like with Keplerian circular orbits (whereas vorticity remains retrograde, even in the inertial frame).

In summary, after decay of the initial three-dimensional turbulence and settling of the solid particles toward the nebula mid-plane, two-dimensional turbulence could persist for a long time and organizes into long-lived vortices able to strongly concentrate the solid material. This allows us to suggest new solutions to some major problems in the modelling of planetary formation:

- (i) the gravitational instability at the origin of the planetesimals is made easier,
- (ii) the cores of the four giant planets form in less than 10^5 yrs, while the terrestrial planets result from longer planetesimal accumulation.

Another important consequence of our model is simply related to the fact that, in a given vortex, particles with $\tau_S \sim 1$ are preferentially captured. This corresponds to dense particles inside the optimal radius r^* and to light ones outside. An efficient mechanism of chemical segregation is therefore provided by the mass/area dependance of the friction parameter; it could help explaining some of the strong disparities observed in the compositions of meteorites and planets.

Of course the existence and structure of our long-lived vortices would require further dynamical justification. However it must be stressed that the capture mechanism we describe is insensitive to the choice of the starting assumptions (i.e) nebula model and constant particle size. Indeed, all the conclusions of the paper hold as long as the friction time τ_S increases with the sun distance, reaching unity at Jupiter's orbit. This is due to the fact that τ_S , which depends on the ratio $\Omega/(\rho_{gas} C_S)$, strongly increases with the distance from the sun, a property which holds for various nebula structures and a wide range of particle sizes, and vortex shapes (provided its core fits with

epicyclic motion). Our model has therefore a strong predictive potentiality, and it is reasonable to consider it as a possible and fruitful alternative to the classical scenario of planetesimal formation.

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